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The perturbative proton form factor reexamined

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Abstract. We recalculate the proton Dirac form factor based on the perturbative QCD factorization theorem, which includes Sudakov suppression. The evolution scale of the proton wave functions and the infrared cutoffs for the Sudakov re-summation are carefully chosen such that the soft divergences from large coupling constants are diminished and perturbative QCD predictions are stabilized. We find that the King–Sachrajda model for the proton wave function leads to results which are in better agreement with experimental data than those from the Chernyak–Zhitnitsky wave function.

1 Introduction

Since the proposal of the improved perturbative QCD (PQCD) factorization formulas for exclusive processes, with the Sudakov re-summation taken into account [1], there have been many applications in the literature, such as the pion form factor [2], photon annihilation into pions [3], the proton form factors [4,5], pion Compton scattering [6], proton–anti-proton annihilation [7], and proton–proton Landshoff scattering [8]. These studies show that in the pion case the nonperturbative contributions from the end points of parton momentum fractions are moderated by Sudakov suppression, and perturbative predictions become relatively reliable. However, in the processes involving protons, because more partons share the external momentum, the infrared divergences associated with soft partons, which appear in hard scattering subamplitudes, are severer. It is then a concern whether Sudakov suppression of the end-point nonperturbative enhancements is strong enough to maintain the applicability of PQCD to the proton form factor at currently accessible energy scales.

The improved factorization formalism has been applied to the proton form factor [4]. However, the choice of the infrared cutoffs for the re-summation was criticized [5]: the end-point enhancements are in fact not diminished completely by Sudakov suppression under the above choice of cutoffs, implying that PQCD predictions remain unreliable. A modified choice of the cutoffs has been proposed [5], and the soft enhancements were found to be suppressed. Unfortunately, it turned out that the PQCD contributions amount to only half of the data, and hence it was concluded that higher-order or higher-twist corrections may be important [5].

In this paper we recalculate the proton Dirac form factor based on the work of [4] by slightly modifying the infrared cutoffs for the re-summation and employing the more complete two-loop expression of the Sudakov factor. It will be shown that the end-point sensitivity is removed, and the PQCD predictions from one of the currently available models of the proton wave function match the experimental data well. We then confirm the applicability of the improved PQCD formalism for momentum transfer around a few GeV. However, we emphasize that the uncertainty involved in our analysis is not diminishingly small, and that the method in [9] based on the overlap integral of the proton wave functions may be regarded as a complementary approach to ours.

2 Factorization

According to the PQCD theory for exclusive processes [10], the proton Dirac form factor can be factorized into two types of subprocesses: wave functions which contain the nonperturbative information of the initial- and final-state protons, and a hard subamplitude which describes the scattering of a valence quark of the proton from the energetic photon. The first of these cannot be calculated perturbatively and needs to be parametrized by a model or to be derived by nonperturbative methods such as QCD sum rules. The second, characterized by a large momentum flow, is calculable in perturbation theory. We quote the factorization formula for the proton form factor derived in [4]:

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$$F_1^p(Q^2) = \int_0^1 (\mathrm{d}x)(\mathrm{d}x')(\mathrm{d}\mathbf{k}_\mathrm{T})(\mathrm{d}\mathbf{k}'_\mathrm{T})\bar{Y}_{\alpha'\beta'\gamma'}(k'_i,P',\mu) \times H_{\alpha'\beta'\gamma'\alpha\beta\gamma}(k_i,k'_i,Q,\mu)Y_{\alpha\beta\gamma}(k_i,P,\mu)\,, \quad (1)$$

where

$$(\mathrm{d}x) = \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}x_3\delta\left(\sum_{i=1}^3 x_i - 1\right)\,,$$
$$(\mathrm{d}\mathbf{k}_\mathrm{T}) = \mathrm{d}\mathbf{k}_{1T}\mathrm{d}\mathbf{k}_{2T}\mathrm{d}\mathbf{k}_{3T}\delta\left(\sum_{i=1}^3 \mathbf{k}_{i\mathrm{T}}\right)\,. \quad (2)$$

$P = (P^+, 0, \mathbf{0})$ is the initial-state proton momentum, and $x_i = k_i^+/P^+$ and $\mathbf{k}_{i\mathrm{T}}$ are the longitudinal momentum fraction and transverse momenta of the parton i, respectively. The primed variables $P' = (0, P'^-, \mathbf{0})$, $x'_i = k'^-_i/P'^-$ and $\mathbf{k}'_{i\mathrm{T}}$ are associated with the final-state proton. $Q^2 = 2P\cdot P'$ is the momentum transfer. In the Breit frame we have $P^+ = P'^- = Q/\sqrt{2}$. The scale μ is the renormalization and factorization scale.

The initial distribution amplitude $Y_{\alpha\beta\gamma}$, defined by the matrix element of three local operators in the axial gauge [11,13], is given by

$$\begin{aligned} Y_{\alpha\beta\gamma} &= \frac{1}{2\sqrt{2}N_\mathrm{c}}\int\prod_{l=1}^2\frac{\mathrm{d}y_l^-\mathrm{d}\mathbf{y}_l}{(2\pi)^3}\exp(\mathrm{i}k_l y_l) \times\epsilon^{abc}\langle 0|T[u_\alpha^a(y_1)u_\beta^b(y_2)d_\gamma^c(0)]|P\rangle \\ &= \frac{f_\mathrm{N}(\mu)}{8\sqrt{2}N_\mathrm{c}}[(\not{P}C)_{\alpha\beta}(\gamma_5 N)_\gamma V(k_i,P,\mu) \\ &\quad +(\not{P}\gamma_5 C)_{\alpha\beta}N_\gamma A(k_i,P,\mu) \\ &\quad -(\sigma_{\mu\nu}P^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5 N)_\gamma T(k_i,P,\mu)]\,, \end{aligned} \quad (3)$$

where $N_\mathrm{c} = 3$ is the color number, $|P\rangle$ the initial proton state, u and d the quark fields, a, b and c the color indices, and α, β and γ the spinor indices. In our notation, 1 and 2 label the two u quarks and 3 labels the d quark. The second form shows the explicit Dirac matrix structure [11], where f_N is the normalization constant [12], N the proton spinor, C the charge conjugation matrix and $\sigma_{\mu\nu} \equiv [\gamma_\mu, \gamma_\nu]/2$. The amplitude $\bar{Y}_{\alpha'\beta'\gamma'}(k'_i, P', \mu)$ for the final-state proton is defined similarly. By using the permutation symmetry [11] and the constraint that the total isospin of the three quarks is equal to 1/2, it can be shown that the three functions V, A, and T are not independent and related to a single function ψ via [11]

$$\begin{aligned} V(k_1,k_2,k_3,P,\mu) &= \frac{1}{2}\left[\psi(k_2,k_1,k_3,P,\mu) + \psi(k_1,k_2,k_3,P,\mu)\right]\,, \\ A(k_1,k_2,k_3,P,\mu) &= \frac{1}{2}\left[\psi(k_2,k_1,k_3,P,\mu) - \psi(k_1,k_2,k_3,P,\mu)\right]\,, \\ T(k_1,k_2,k_3,P,\mu) &= \frac{1}{2}\left[\psi(k_1,k_3,k_2,P,\mu) + \psi(k_2,k_3,k_1,P,\mu)\right]\,. \end{aligned} \quad (4)$$

The hard subamplitude $H_{\alpha'\beta'\gamma'\alpha\beta\gamma}$ is obtained from the photon–quark scattering diagrams, and the expressions for the integrands $\bar{Y}_{\alpha'\beta'\gamma'}H_{\alpha'\beta'\gamma'\alpha\beta\gamma}Y_{\alpha\beta\gamma}$ are from Table I in [4]. Employing a series of permutations of the parton kinematic variables, (1) in Fourier transform space reduces to

$$F_1^p(Q^2) = \sum_{j=1}^2\frac{8\pi^2}{27}\int_0^1(\mathrm{d}x)(\mathrm{d}x')(\mathrm{d}\mathbf{b})[f_\mathrm{N}(\mu)]^2 \times\tilde{H}_j(x_i,x'_i,\mathbf{b}_i,Q,\mu) \times\Psi_j(x_i,x'_i,\mathbf{b}_i,P,P',\mu)\,, \quad (5)$$

with $\mathbf{b}_i$ the conjugate variable to $\mathbf{k}_{i\mathrm{T}}$ and $(\mathrm{d}\mathbf{b}) = \mathrm{d}\mathbf{b}_1\mathrm{d}\mathbf{b}_2/(2\pi)^4$. The explicit expressions for $\tilde{H}_j$ and Ψ_j in terms of ψ are given below.

3 Sudakov suppression

The Sudakov re-summation of the leading (double) and next-to-leading (single) logarithms in ψ leads to

$$\begin{aligned} &\psi(x_i,\mathbf{b}_i,P,\mu) \\ &= \exp\left[-\sum_{l=1}^3 s\left(x_l,\frac{1}{w},Q\right)\right. \\ &\left.-3\int_w^\mu\frac{\mathrm{d}\bar{\mu}}{\bar{\mu}}\gamma_\mathrm{q}\left(\alpha_s(\bar{\mu})\right)\right]\phi(x_i,w)\,, \end{aligned} \quad (6)$$

where the quark anomalous dimension $\gamma_\mathrm{q}(\alpha_s) = -\alpha_s/\pi$ in the axial gauge governs the renormalization-group (RG) evolution of ψ. The function ϕ, obtained by factoring the Q dependence out of ψ, corresponds to the standard parton model. The exponent s is written as [13]

$$s\left(x,\frac{1}{w},Q\right) = \int_w^{xQ/\sqrt{2}}\frac{\mathrm{d}p}{p} \times\left[\ln\left(\frac{xQ}{\sqrt{2}p}\right)A\left(\alpha_s(p)\right) + B\left(\alpha_s(p)\right)\right]\,, \quad (7)$$

where the anomalous dimensions A to two loops and B to one loop are

$$\begin{aligned} A &= \mathcal{C}_F\frac{\alpha_s}{\pi} + \left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_\mathrm{f} + \frac{8}{3}\beta_0\ln\left(\frac{\mathrm{e}^{\gamma_\mathrm{E}}}{2}\right)\right]\left(\frac{\alpha_s}{\pi}\right)^2, \\ B &= \frac{2}{3}\frac{\alpha_s}{\pi}\ln\left(\frac{\mathrm{e}^{2\gamma_\mathrm{E}-1}}{2}\right)\,, \end{aligned} \quad (8)$$

$n_\mathrm{f} = 3$ being the flavor number and γ_E the Euler constant. The two-loop running coupling constant,

$$\frac{\alpha_s(\mu)}{\pi} = \frac{1}{\beta_0\ln(\mu^2/\Lambda^2)} - \frac{\beta_1}{\beta_0^3}\frac{\ln\left[\ln(\mu^2/\Lambda^2)\right]}{\ln^2(\mu^2/\Lambda^2)}\,, \quad (9)$$

with the coefficients

$$\beta_0 = \frac{33-2n_\mathrm{f}}{12}\,, \quad \beta_1 = \frac{153-19n_\mathrm{f}}{24}\,, \quad (10)$$

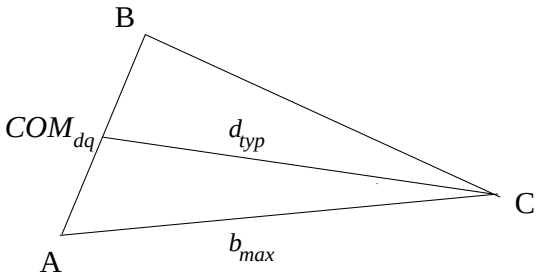

Fig. 1. The typical transverse distance, $d_{\rm typ}$. The transverse distance between the quarks A and B is the smallest among the three quarks. The diquark constituents are therefore considered to be the quarks A and B. The center of mass of the diquark, $COM_{\rm q}$, is taken to be the central point of the line that connects these two quarks. $d_{\rm typ}$ is then defined as the distance between $COM_{\rm q}$ and the third quark C

and the QCD scale $\Lambda \equiv \Lambda_{\rm QCD}$, will be substituted into (7).

The infrared cutoff w is chosen to be the inverse of a typical transverse distance among the three valence quarks. We try different definitions of this cutoff to determine its influence on the final result. One possible choice is $w = 1/b_{\rm max}$, $b_{\rm max} = \max(b_l)$, $l = 1, 2, 3$, as adopted in [5], with $b_3 = |\mathbf{b}_1 - \mathbf{b}_2|$. As long as all of these mass scales are much larger than Λ, the Sudakov form factor should not give any suppression. As one of these scales gets close to Λ, the Sudakov form factor tends to zero and suppresses this region. We find that choosing the infrared cutoff in this fashion suppresses all the infrared divergences and leads to a self-consistent calculation of the form factor. However, this choice does not always correspond to a typical size of the three-quark system. A more appropriate definition is obtained by considering it as a quark–diquark like configuration. The diquark constituents are taken to be those two quarks that are closest to each other in the transverse plane. We now define the typical transverse distance, $d_{\rm typ}$, as the distance between the center of mass of the diquark and the remaining third quark (Fig. 1).

This is clearly a more reasonable measure of the distance in the three-quark system that can be resolved by a gluon. We shall therefore take the infrared cutoff as cw, c being a parameter that is allowed to deviate slightly from unity. When we put $c = 1$, we recover the original choice of the cutoff. Next, c is chosen such that for a large number of randomly chosen triangles, as the one in Fig. 1, we get on average $\langle d_{\rm typ}/b_{\rm max}\rangle = 1/c$. Defining c in such a way, gives $c \approx 1.14$. The introduction of this parameter c is also natural from the viewpoint of the re-summation, since the scale cw, with c of order unity, is equivalent to w within the accuracy of next-to-leading logarithms [13].

We find that both of these choices of the cutoff lead to self-consistent calculations of the form factor in the sense that the form factor saturates at the large distance cutoff b_c. Remarkably, we find that with the small modification of w into cw, which differs from what was used in [5], the results are in good agreement with experimental data. The dependence of the final results on the precise choice of cw shows that large distance contributions cannot be completely dismissed, and give a contribution of about 25%–50% at laboratory energies. Nevertheless, we find it encouraging that a physically motivated cutoff gives good agreement with experiments.

The choice of scales for the Sudakov re-summation in (6) is compared to that adopted in [4], where the different cutoffs b_l are assigned to each exponent s and to each integral involving $\gamma_{\rm q}$:

$$\begin{aligned}&\psi(x_i, \mathbf{b}_i, P, \mu)\\ &= \exp\left[-\sum_{l=1}^{3}\left(s(x_l, b_l, Q) + \int_{\frac{1}{b_l}}^{\mu}\frac{\mathrm{d}\bar{\mu}}{\bar{\mu}}\gamma_q\left(\alpha_s(\bar{\mu})\right)\right)\right]\\ &\times\phi(x_i, w)\ . \end{aligned} \tag{11}$$

The Sudakov factor in (11) does not suppress the soft divergences from $b_l \to 1/\Lambda$ completely. For example, the divergences from $b_1 \to 1/\Lambda$, which appear in $\phi(x_i, w)$ at $w \to \Lambda$ (see Sect. 4.), survive as $x_1 \to 0$, since $s(x_1, b_1, Q)$ vanishes and $s(x_2, b_2, Q)$ and $s(x_3, b_3, Q)$ remain finite. On the other hand, w should play the role of the factorization scale, above which QCD corrections give the perturbative evolution of the wave function ψ in (6), and below which QCD corrections are absorbed into the initial condition ϕ. It is then not reasonable to choose the cutoffs b_l for the Sudakov re-summation different from w.

4 RG evolution

The RG evolution of the hard scattering subamplitudes is written as

$$\begin{aligned}&\tilde{H}_j(x_i, x_i', \mathbf{b}_i, Q, \mu)\\ &= \exp\left[-3\sum_{l=1}^{2}\int_{\mu}^{t_{jl}}\frac{\mathrm{d}\bar{\mu}}{\bar{\mu}}\,\gamma_{\rm q}\left(\alpha_s(\bar{\mu})\right)\right]\\ &\times\tilde{H}_j(x_i, x_i', \mathbf{b}_i, Q, t_{j1}, t_{j2})\ , \end{aligned} \tag{12}$$

where the explicit expressions of t are

$$\begin{aligned}t_{11} &= \max\left[\sqrt{(1-x_1)(1-x_1')}Q, 1/b_1\right]\ ,\\ t_{21} &= \max\left[\sqrt{x_1x_1'}Q, 1/b_1\right]\ ,\\ t_{12} = t_{22} &= \max\left[\sqrt{x_2x_2'}Q, 1/b_2\right]\ . \end{aligned} \tag{13}$$

The first scales in the brackets are associated with the longitudinal momenta of the exchanged gluons and the second scales with the transverse momenta. The arguments t_{j1} and t_{j2} of $\tilde{H}_j$ denote that each α_s is evaluated at the largest mass scale of the corresponding gluon.

Inserting (6) and (12) into (5), we obtain

$$\begin{aligned}F_1^p(Q^2) &= \sum_{j=1}^{2}\frac{4\pi}{27}\int_0^1(\mathrm{d}x)(\mathrm{d}x')\\ &\times\int_0^{\infty}b_1\mathrm{d}b_1b_2\mathrm{d}b_2\int_0^{2\pi}\mathrm{d}\theta[f_{\rm N}(cw)]^2\\ &\times\tilde{H}_j(x_i, x_i', b_i, Q, t_{j1}, t_{j2})\,\Psi_j(x_i, x_i', cw)\\ &\times\exp\left[-S(x_i, x_i', cw, Q, t_{j1}, t_{j2})\right]\ , \end{aligned} \tag{14}$$

with

$$\tilde{H}_1 = \frac{2}{3}\alpha_s(t_{11})\alpha_s(t_{12}) \times K_0\left(\sqrt{(1-x_1)(1-x_1')}Qb_1\right) \times K_0\left(\sqrt{x_2x_2'}Qb_2\right) ,$$
$$\tilde{H}_2 = \frac{2}{3}\alpha_s(t_{21})\alpha_s(t_{22}) \times K_0\left(\sqrt{x_1x_1'}Qb_1\right) K_0\left(\sqrt{x_2x_2'}Qb_2\right) . \quad (15)$$

The variable θ is the angle between $\mathbf{b}_1$ and $\mathbf{b}_2$. K_0 is the modified Bessel function of order zero. The expressions for Ψ_j are

$$\Psi_1 = \frac{2(\phi\phi')_{123} + 8(TT')_{123} + 2(\phi\phi')_{132}}{(1-x_1)(1-x_1')} + \frac{8(TT')_{132} - (\phi\phi')_{321} - (\phi\phi')_{231}}{(1-x_1)(1-x_1')} ,$$

$$\Psi_2 = \frac{2(\phi\phi')_{132} - 2(TT')_{123}}{(1-x_2)(1-x_1')} + \frac{(\phi\phi')_{123} - 8(TT')_{132} - 2(\phi\phi')_{321}}{(1-x_3)(1-x_1')} , \quad (16)$$

which group together the products of the initial and final wave functions in the notation

$$(\phi\phi')_{123} \equiv \phi(x_1, x_2, x_3, cw)\phi(x_1', x_2', x_3', cw) . \quad (17)$$

(TT') is defined similarly based on (4) but with ψ replaced by ϕ. The Sudakov exponent S is given by

$$S(x_i, x_i', cw, Q, t_{j1}, t_{j2}) = \sum_{l=1}^{3} s(x_l, cw, Q) + 3\int_{cw}^{t_{j1}} \frac{\mathrm{d}\bar{\mu}}{\bar{\mu}}\gamma_q\left(\alpha_s(\bar{\mu})\right) + \sum_{l=1}^{3} s(x_l', cw, Q) + 3\int_{cw}^{t_{j2}} \frac{\mathrm{d}\bar{\mu}}{\bar{\mu}}\gamma_{\mathrm{q}}\left(\alpha_s(\bar{\mu}^2)\right) . \quad (18)$$

For the wave function ϕ, we will consider both the Chernyak–Zhitnitsky (CZ) model [11] and King–Sachrajda (KS) model [14]. They are decomposed in terms of the first six Appel polynomials $A_j(x_i)$, which are eigensolutions of the evolution equation for the nucleon wave function [10, 15]

$$\phi(x_i, w) = \phi_{\mathrm{as}}(x_i) \times \sum_{j=0}^{5} N_j \left[\frac{\alpha_s(w)}{\alpha_s(\mu_0)}\right]^{b_j/(4\beta_0)} a_j A_j(x_i) , \quad (19)$$

with $\mu_0 \approx 1$ GeV. The constants N_j, a_j and b_j are given in Table 1. $\phi_{\mathrm{as}}(x_i) = 120x_1x_2x_3$ is the asymptotic form of ϕ. The evolution of the dimensional constant f_{N} is given by

$$f_{\mathrm{N}}(w) = f_{\mathrm{N}}(\mu_0)\left[\frac{\alpha_s(w)}{\alpha_s(\mu_0)}\right]^{1/(6\beta_0)} , \quad (20)$$

with $f_{\mathrm{N}}(\mu_0) = (5.2 \pm 0.3) \times 10^{-3}$ GeV^2 [11].

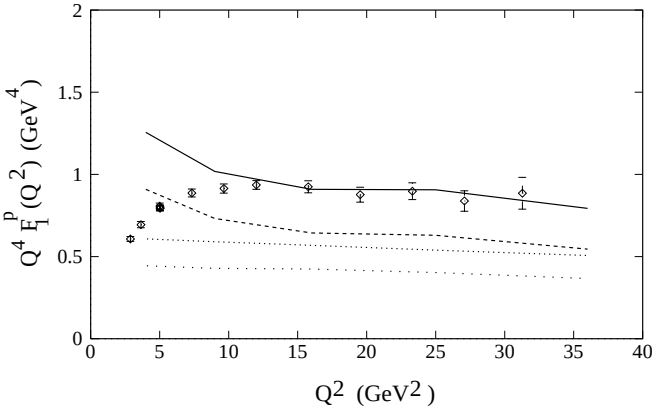

Fig. 2. Dependence of $Q^4F_1^p$ on Q^2 for the use of the KS wave function (solid line, $c = 1.14$;dense-dot line, $c = 1$) and for the CZ wave function (dashed line, $c = 1.14$; dotted line, $c = 1$). The experimental data with error bars are also shown

5 Numerical results

In order to calculate the seven-dimensional integral (14), we use the VEGAS Monte Carlo routine [16]. We set the factor $\exp[-s(\xi, cw, Q)]$ to unity whenever $\xi Q/\sqrt{2} < cw$, since in this small b region higher-order corrections should be absorbed into the hard scattering [1], instead of into the wave function, giving its evolution. Similarly, we set the Sudakov factor $\exp(-S)$ to unity in the small b region where it includes a small enhancement. As cw approaches Λ, the Sudakov factor vanishes, implying that the whole integrand of (14) also vanishes.

First we choose the parameter value $c = 1$. The results of $Q^4F_1^p$ for $\Lambda = 0.2$ GeV from the use of the KS wave function, along with the experimental data [17,18], are shown in Fig. 2. The PQCD predictions amount only to about half of the data. It is then possible that higher-order or higher-Fock-state contributions are important for the explanation of the data, which are certainly worthy of further studies. However, before jumping to that conclusion, we investigate the effect from the freedom of varying the parameter c. The results with $c = 1.14$ are also displayed in Fig. 2. It is found that our predictions match the data well. Note that varying c makes a difference in the re-summation at the level of next-to-leading logarithms, which can be regarded as an uncertainty of our formalism. Therefore, we argue that the current data can be explained within the uncertainty of our approach.

Following [4], we should analyze how the contributions to $Q^4F_1^p$ are distributed in the b_1-b_2 plane. The integration is done with both variables b_1 and b_2 cut off at a common value b_{c}. If the perturbative region dominates, most of the contributions will be quickly accumulated below a small b_{c}. The numerical outcomes (with $c = 1.14$) are shown in Fig. 3. All the curves, showing the dependence of $Q^4F_1^p$ on b_{c}, increase from the origin and reach their full height at $b_{\mathrm{c}} = 0.9/\Lambda$. The curves exhibit small humps at the high end of b_{c}, which imply that the evolution of the wave function gives a small negative contribution in the large b region. A standard of self-consistency is that 50% of the

Table 1. Appel polynomial coefficients in (19) for the nucleon wave function $\phi(x_i, w)$ of the CZ and KS models [11,14] with the scale $\mu_0 \approx 1$ GeV [17]

j	a_j(CZ)	a_j(KS)	N_j	b_j	$A_j(x_i)$
0	1.00	1.00	1	0	1
1	0.410	0.310	21/2	20/9	$x_1 - x_3$
2	−0.550	−0.370	7/2	24/9	$2 - 3(x_1 + x_3)$
3	0.357	0.630	63/10	32/9	$2 - 7(x_1 + x_3) + 8(x_1^2 + x_3^2) + 4x_1x_3$
4	−0.0122	0.00333	567/2	40/9	$x_1 - x_3 - (4/3)(x_1^2 - x_3^2)$
5	0.00106	0.0632	81/5	42/9	$2 - 7(x_1 + x_3) + 14x_1x_3$ $+(14/3)(x_1^2 + x_3^2)$

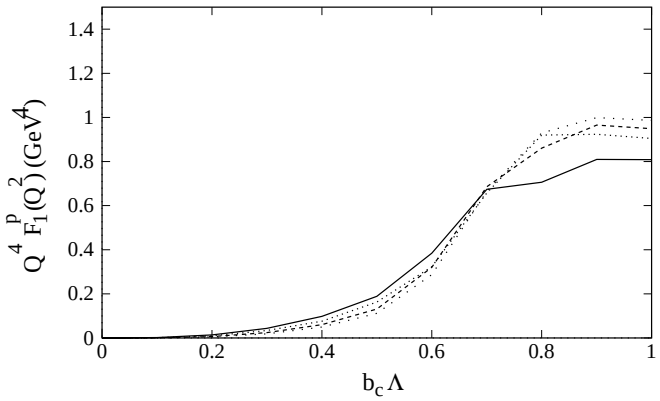

Fig. 3. Dependence of $Q^4 F_1^p$ on the cutoff b_c with the KS wave function for $Q^2 = 12$ GeV^2 (dotted line), $Q^2 = 16$ GeV^2 (dashed line), $Q^2 = 25$ GeV^2 (dense-dot line), and $Q^2 = 36$ GeV^2 (solid line)

whole amount of $Q^4 F_1^p$ is accumulated from the region with α_s/π smaller than 0.5. Based on this standard, the results with $Q^2 > 10$ GeV^2 are reliable. Therefore, the applicability of PQCD to the proton form factor at currently accessible energy scale $Q^2 \approx 35$ GeV^2 is justified.

The CZ wave function is also employed, and the corresponding results are shown in Fig. 2. It is observed that the values are only about 2/3 and 3/4 of those derived from the KS model with $c = 1.14$ and $c = 1$, respectively, and are far below the data. Hence, we claim that the KS proton wave function is more phenomenologically appropriate.

6 Conclusion

In this work we have modified the choice of the infrared cutoffs for the re-summation, and employed the more complete two-loop expression of the Sudakov factor compared to the previous analyses. With these modifications, we have been able to explain self-consistently the experimental data of the proton Dirac form factor for $Q^2 > 10$ GeV^2. It has been found that the KS wave function is phenomenologically more appropriate than the CZ model. Though the coupling constant α_s is not so small that we could consider the perturbative results as exact, the nonperturbative region denoted by $b \to 1/\Lambda$ does become less important in our analysis as shown in Fig. 3. A complementary study based on nonperturbative approaches such as QCD sum rules and the determination of the transition scale of the proton form factor to PQCD, as performed in [6], may provide further justification. The contributions from higher Fock states should also be investigated, which may be important in the intermediate energy range.

Compared to the case of the pion form factor [1], the uncertainty at the level of next-to-leading logarithms is more serious here, indicating that the leading logarithms collected by the suppressing factor $\exp(-s)$ are not strong enough in the proton case. Therefore, higher-order corrections to the evolution of the wave function need to be computed in order to extract the best choice of the parameter c. In this paper, however, we use a physically motivated definition of c along with the KS wave function. The uncertainty in this parameter can be absorbed into the universal proton wave function, which can then be used to study other QCD processes consistently. This viewpoint can be checked by applying our formalism to other processes involving proton in the future.

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References

1. H-n. Li and G. Sterman, Nucl. Phys. B **381** (1992) 129
2. T. Gousset and B. Pire, Phys. Rev. D **51** (1995) 15; R. Jakob and P. Kroll, Phys. Lett. B **315** (1993) 463
3. I.V. Musatov and A.V. Radyushkin, Preprint No. JLAB-Thy-97-07 (hep-ph/9702443)
4. H-n. Li, Phys. Rev. D **48** (1993) 4243
5. R. Jakob, P. Kroll, M. Bergmann, and N.G. Stefanis, Z. Phys. C **66** (1995) 267

6. C. Corianò and H-n. Li, Phys. Lett. B **309** (1993) 409; Nucl. Phys. B **434** (1995) 535
7. T. Hyer, Phys. Rev. D **47** (1993) 3875
8. M.G. Sotiropoulos and G. Sterman, Nucl. Phys. B **419** (1994) 77
9. J. Bolz and P. Kroll, Preprint No. WU B 95-35 (hep-ph/9603289)
10. G.P. Lepage and S.J. Brodsky, Phys. Rev. Lett. **43** (1979) 545; Phys. Rev. D **22**, (1980) 2157
11. V.L. Chernyak and A.R. Zhitnitsky, Yad. Fiz. **31** (1980) 1053 [Sov. J. Nucl. Phys. 31 (1980) 544]; Nucl. Phys. B **216** (1983) 373; B **246** (1984) 52; Phys. Rep. **112** (1984) 173
12. B.L. Ioffe, Nucl. Phys. B **188** (1981) 317; B **191** (1981) 591
13. J. Botts and G. Sterman, Nucl. Phys. B **325** (1989) 62
14. I.D. King and C.T. Sachrajda, Nucl. Phys. B **279** (1987) 785
15. S.J. Brodsky and G.P. Lepage, Phys. Scr. **23** (1981) 945
16. W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery, *Numerical Recipes in Fortran*, 2nd ed. (Cambridge University Press, Cambridge, 1992)
17. C.R. Ji, A.F. Sill and R.M. Lombard-Nelsen, Phys. Rev. D **36** (1987) 165
18. G. Arnold et al., Phys. Rev. Lett. **57** (1986) 174